

by checking, by means of equation (2), the diffraction conditions in both cases*.

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A Theoretical Calculation of X-ray Absorption Cross Sections

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(Received 9 November 1965 and in revised form 12 September 1966)

Numerical values of X-ray absorption cross-sections for atoms are given for 10 commonly used X-ray wavelengths. The calculations are based on a multipole expansion of the appropriate matrix elements, using hydrogen-like eigenfunctions, for the *K*, *L*, *M* and *N* electrons. For the given figures, agreement with experiment is in general better than 5%.

Photoelectric absorption cross-sections for X-rays have been calculated by several authors using hydrogen-like eigenfunctions (Bethe & Salpeter, 1957). These calculations have been made for the *K*- and *L*-shells using the dipole approximation. Recently one of us discussed the angular dependence of the Borrman effect, and it was found that apart from the influence of thermal vibrations within the crystal, this angular dependence is caused by electrical quadrupole transitions (Wagenfeld, 1966).* Here the methods, and to some extent the results of Hönl's (1933) and Eisenlohr & Müller's (1954) calculations of the generalized atomic scattering factor for the region of anomalous dispersion have been used, but the calculations have been extended for the *M*- and *N*-shells as well. A further complication has been the inaccuracy of the screening constants which are needed for the hydrogen-like eigenvalues.

* Dipole-octupole transitions, which give a smaller contribution than quadrupole transitions, are also included. It can be proved that electrical octupole transitions are very weak. Magnetic transitions do not occur in this case.

Since Bearden (1964) has measured, or given best values for, the X-ray wave length for most elements with high precision, one can determine the screening constants from his data, using Sommerfeld's fine structure formulae for the term values. Since the calculations are based on hydrogen-like eigenfunctions, the eigenvalues for the energies do not agree exactly with the energies of the measured absorption edges. Hence one cannot expect that this calculation will hold for the wavelength region near to and below an absorption edge. In the following tables we therefore publish only the numerical values for the atomic absorption cross-sections which we believe are theoretically justified. For this reason we also do not give cross-sections for the wave length region longer than the *L*-shell absorption edge. However, we compared some of the values with experimental data, and found agreement within 5%. The comparisons showed that these values are significantly better than the theoretical values given in *International Tables for X-ray Crystallography* (1962).

Table 1 (cont.)

AT. NO	WAVELENGTH	Cr $K\alpha_1$, 2.2897 Å	Cr $K\beta_1$, 2.0849 Å	Fe $K\alpha_1$, 1.9360 Å	Fe $K\beta_1$, 1.7566 Å	Cu $K\alpha_1$, 1.5406 Å	Cu $K\beta_1$, 1.3922 Å	Mo $K\alpha_1$, 0.7093 Å	Mo $K\beta_1$, 0.6323 Å	Ag $K\alpha_1$, 0.5594 Å	Ag $K\beta_1$, 0.4975 Å
41	DIPOLE	67.660	52.061	42.285	32.155	22.187	16.643	2.404		6.790	
	QUADRUPOLE	0.038	0.041	0.044	0.048	0.053	0.055	0.041		0.173	
	TOTAL	67.698	52.102	42.329	32.203	22.240	16.697	2.445		6.982	
42	DIPOLE	74.480	57.345	46.600	35.459	24.487	18.378	2.664	1.911	7.388	
	QUADRUPOLE	0.037	0.039	0.042	0.046	0.051	0.053	0.043	0.039	0.169	
	TOTAL	74.516	57.384	46.642	35.505	24.538	18.432	2.707	1.950	7.557	
43	DIPOLE	63.107	51.299	39.050	26.981	20.259	2.945	2.113			
	QUADRUPOLE	0.038	0.040	0.044	0.049	0.053	0.055	0.045	0.041		
	TOTAL	63.144	51.338	39.094	29.030	20.312	2.990	2.154			
44	DIPOLE	68.969	56.093	42.727	29.574	22.199	3.238	2.325			
	QUADRUPOLE	0.037	0.038	0.042	0.047	0.051	0.057	0.047	0.043		
	TOTAL	69.006	56.131	42.769	29.594	22.250	3.285	2.368			
45	DIPOLE	61.452	46.840	32.419	24.372	3.568	2.564	1.801			
	QUADRUPOLE	0.037	0.040	0.045	0.049	0.051	0.058	0.045	0.040		
	TOTAL	61.489	46.880	32.464	24.421	3.617	2.608	1.841			
46	DIPOLE	67.186	51.242	35.494	26.699	3.923	2.820	1.982			
	QUADRUPOLE	0.037	0.038	0.043	0.048	0.050	0.056	0.046	0.042		
	TOTAL	67.223	51.281	35.537	26.747	3.973	2.867	2.024			
47	DIPOLE	56.241	38.999	29.360	4.134	3.118	2.193	1.564			
	QUADRUPOLE	0.037	0.041	0.046	0.051	0.058	0.064	0.044	0.040		
	TOTAL	56.278	39.040	29.406	4.186	3.166	2.237	1.604			
48	DIPOLE	61.192	42.468	31.992	4.740	3.412	2.402	1.714			
	QUADRUPOLE	0.037	0.040	0.044	0.053	0.053	0.059	0.046	0.041		
	TOTAL	61.229	42.508	32.036	4.793	3.462	2.447	1.755			
49	DIPOLE	46.157	34.793	5.174	3.727	2.625	1.874				
	QUADRUPOLE	0.038	0.042	0.048	0.054	0.051	0.047	0.043	0.040		
	TOTAL	46.195	34.835	37.793	5.228	3.778	2.672	1.917			
50	DIPOLE	50.050	37.753	5.637	4.062	2.863	2.045				
	QUADRUPOLE	0.038	0.040	0.044	0.053	0.052	0.049	0.045	0.042		
	TOTAL	50.087	37.793	5.691	4.114	2.911	2.090				
51	DIPOLE	54.196	40.905	6.129	4.419	3.116	2.228				
	QUADRUPOLE	0.037	0.039	0.055	0.053	0.053	0.053	0.046	0.046		
	TOTAL	54.234	40.944	6.184	4.473	3.166	2.274				
52	DIPOLE	44.322	6.668	4.811	3.395	2.428					
	QUADRUPOLE	0.038	0.056	0.056	0.055	0.052	0.052	0.048	0.048		
	TOTAL	44.360	6.724	4.866	3.447	2.476					
53	DIPOLE	47.506	7.189	5.191	3.666	2.624					
	QUADRUPOLE	0.038	0.056	0.056	0.055	0.053	0.053	0.049	0.049		
	TOTAL	47.543	7.245	5.247	3.719	2.674					
54	DIPOLE	7.823	5.651	3.992	2.859						
	QUADRUPOLE	0.056	0.056	0.056	0.054	0.054	0.054	0.051	0.051		
	TOTAL	7.879	5.707	4.046	2.909						
55	DIPOLE	8.402	6.072	4.292	3.025						
	QUADRUPOLE	0.056	0.056	0.056	0.055	0.055	0.055	0.052	0.052		
	TOTAL	8.458	6.128	4.346	3.127						
56	DIPOLE	9.072	6.560	4.600	3.327						
	QUADRUPOLE	0.056	0.057	0.057	0.056	0.056	0.056	0.053	0.053		
	TOTAL	9.127	6.617	4.696	3.380						
57	DIPOLE	9.751	7.057	4.995	3.583						
	QUADRUPOLE	0.055	0.057	0.057	0.056	0.056	0.056	0.054	0.054		
	TOTAL	9.806	7.113	5.051	3.638						
58	DIPOLE	10.459	7.572	5.361	3.848						
	QUADRUPOLE	0.054	0.056	0.056	0.055	0.055	0.055	0.053	0.053		
	TOTAL	10.513	7.623	5.418	3.903						
59	DIPOLE	11.229	8.133	5.762	4.137						
	QUADRUPOLE	0.053	0.056	0.056	0.057	0.057	0.056	0.054	0.054		
	TOTAL	11.282	8.189	5.818	4.193						
60	DIPOLE	12.063	8.742	6.197	4.452						
	QUADRUPOLE	0.053	0.056	0.056	0.057	0.057	0.056	0.054	0.054		
	TOTAL	12.115	8.798	8.798	6.254	4.509					

Table 1 (cont.)

AT. NO.	WAVELENGTH	Mo K α_1 , 0.7093 Å	Mo K β , 0.6323 Å	Ag K α_1 , 0.5594 Å	Ag K β , 0.4975 Å
61	DIPOLE	12.910	9.362	6.640	4.273
	QUADRUPOLE	0.051	0.055	0.057	0.057
	TOTAL	12.962	9.416	6.697	4.830
62	DIPOLE	13.790	10.005	7.101	5.107
	QUADRUPOLE	0.050	0.054	0.056	0.057
	TOTAL	13.840	10.059	7.157	5.164
63	DIPOLE	14.758	10.713	7.608	5.475
	QUADRUPOLE	0.049	0.053	0.056	0.057
	TOTAL	14.807	10.767	7.664	5.532
64	DIPOLE	15.645	11.363	8.072	5.812
	QUADRUPOLE	0.047	0.052	0.055	0.057
	TOTAL	15.692	11.414	8.128	5.869
65	DIPOLE	16.655	12.103	8.604	6.198
	QUADRUPOLE	0.045	0.050	0.054	0.057
	TOTAL	16.700	12.153	8.658	6.254
66	DIPOLE	17.668	12.846	9.137	6.585
	QUADRUPOLE	0.044	0.049	0.053	0.056
	TOTAL	17.712	12.895	9.190	6.641
67	DIPOLE	18.693	13.597	9.675	6.976
	QUADRUPOLE	0.042	0.047	0.052	0.055
	TOTAL	18.735	13.644	9.727	7.031
68	DIPOLE	19.925	14.503	10.327	7.451
	QUADRUPOLE	0.041	0.046	0.051	0.055
	TOTAL	19.965	14.548	10.378	7.506
69	DIPOLE	20.861	15.195	10.827	7.817
	QUADRUPOLE	0.041	0.046	0.052	0.056
	TOTAL	20.902	15.241	10.879	7.873
70	DIPOLE	22.493	16.399	11.696	8.452
	QUADRUPOLE	0.039	0.044	0.049	0.054
	TOTAL	22.532	16.442	11.747	8.506
71	DIPOLE	23.643	17.244	12.304	8.895
	QUADRUPOLE	0.038	0.042	0.047	0.052
	TOTAL	23.682	17.286	12.351	8.947
72	DIPOLE	25.039	18.274	13.047	9.438
	QUADRUPOLE	0.038	0.041	0.046	0.051
	TOTAL	25.078	18.314	13.093	9.489
73	DIPOLE	19.289	13.780	9.973	
	QUADRUPOLE	0.040	0.045	0.050	
	TOTAL	19.328	13.825	10.023	
74	DIPOLE	20.350	14.517	10.531	
	QUADRUPOLE	0.039	0.043	0.049	
	TOTAL	20.389	14.590	10.583	
75	DIPOLE	21.497	15.377	11.142	
	QUADRUPOLE	0.039	0.042	0.047	
	TOTAL	21.535	15.419	11.189	
76	DIPOLE	22.656	16.216	11.756	
	QUADRUPOLE	0.038	0.041	0.046	
	TOTAL	22.695	16.257	11.802	
77	DIPOLE	17.007	12.343		
	QUADRUPOLE	0.040	0.045		
	TOTAL	17.047	12.388		
78	DIPOLE	17.973	13.045		
	QUADRUPOLE	0.039	0.043		
	TOTAL	18.012	13.088		
79	DIPOLE	18.894	13.720		
	QUADRUPOLE	0.039	0.043		
	TOTAL	18.932	13.762		
80	DIPOLE	19.835	14.413		
	QUADRUPOLE	0.038	0.041		
	TOTAL	19.873	14.454		
81	DIPOLE	14.872			
	QUADRUPOLE	0.041			
	TOTAL	14.912			
82	DIPOLE	15.906			
	QUADRUPOLE	0.039			
	TOTAL	15.945			
83	DIPOLE	16.656			
	QUADRUPOLE	0.039			
	TOTAL	16.694			
84	DIPOLE	17.491			
	QUADRUPOLE	0.038			
	TOTAL	17.530			

For a crystal composed of n_i different atoms we obtain the linear absorption coefficient from the formula

$$\tau = \sum_{i=1}^{n_i} N_i \sigma_i ,$$

where N_i is the number of atoms of the species i per cm³, so that

$$N = \sum_{i=1}^{n_i} N_i$$

is the total number of atoms per cm³. σ_i is the photo-electric absorption cross-section for the i th atom.

The numerical values as given in Table 1 are the atomic absorption cross-sections, split into three parts, for ten commonly used X-ray wavelengths.[†] For each element whose atomic number is given in the first column, the contribution to the cross-section of the dipole, dipole-octupole and Compton scattering terms is given in the row labelled *Dipole*. The cross-section due to the quadrupole term is given in the row labelled *Quadrupole*. The sum of these two, which is of course the total cross-section, is given in the row labelled *Total*.

The authors would like to thank Professor J. M. Cowley for helpful discussions.

A University of Melbourne Research Grant is acknowledged by one of us (A.J.G.).

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[†] The values used here are those given in Bearden's (1964) tables of X-ray wavelengths.